

Fig. 1 Force as a function of the distance between the meter and the graphite disk

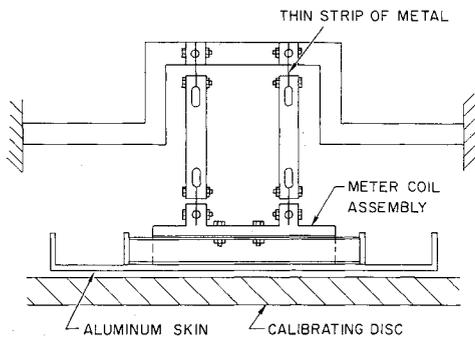


Fig. 2 A displacement parallel to the disk caused by the force on the coils

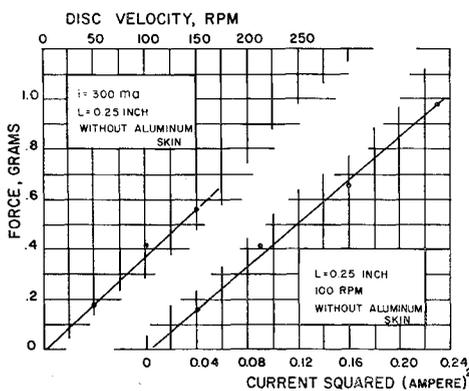


Fig. 3. Linear variation of force with velocity and with the square of the current in the primary coils

periment.³ Results of the measurement are shown in Fig. 1. The calculated and measured forces ($i = 460$ ma with aluminum skin) agree within a factor of 4. The calculated force and measured force curves show nearly identical slopes.

The predicted dependence of the force on u and i^2 shows excellent agreement with the experimentally determined dependence, as can be seen in Fig. 3. The force on the meter in flight would have been approximately 0.006 g. Thus, the presence of a conductivity meter is very unlikely to perturb the trajectory of the re-entry vehicle.

References

¹ Fuhs, A. E., "Development of a device for measuring electrical conductivity of ionized air during re-entry," Physical Research Lab., Space Technology Labs. Inc., STL/TR-60-0000-09256 (September 20, 1960).
² Betchov, R., Fuhs, A. E., Meyer, R. X., and Schaffer, A. B., "Measurement of electrical conductivity of ionized air during re-entry," Physical Research Lab., Aerospace Corp. Rept. TDR-594(1215-01)TR-1 (July 10, 1961); also Aerospace Eng. 21, 54-55, 68-78 (November 1962).
³ Schaffer, A. B., "Experimental study of an airfoil with a boundary layer subjected to magnetoaerodynamic forces," Physical Research Lab., Space Technology Labs. Inc., STL/TR-60-0000-09104 (May 3, 1960).

Behavior of a Trailing Vortex in a Favorable Pressure Gradient

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The purpose of this study is the investigation of a streamwise vortex in low-turbulence channel flow, with particular attention to the high-shear region of the vortex core. Transverse-velocity profiles through the flow indicate that nearly discrete cylindrical vortex sheets are formed.

THE study is confined to vorticity in channel flow because of the relative simplicity of instrumenting the channel. The wind tunnel consisted of a settling chamber and a $3 \times 3 \times 27$ -in. test section with a free-jet exhaust. With an area ratio of 24:1 between settling chamber and test section, the turbulence level was held to an upper limit of 0.06%. The tunnel was driven from the downstream end to a maximum speed of 16 fps.

The vorticity was produced by the following arrangement. Two flat-plate wings at equal but opposite angles of attack were inserted into the flow from opposite sides of the test section, with provision for adjusting tip separation and angles of attack. The resulting tip vortices were forced together by the pressure field to produce a flow that is physically indistinguishable from a single vortex. (This statement was tested successfully by comparing velocity profiles through the vortex in the xy and xz planes.) The wing-tip separation was 0.125 in. and thus of the order of the boundary layer thickness at the wing trailing edge. During the test the Reynolds number based on wing chord lay in the range of 2000 to 17,000.¹

Velocity profiles across the test section were obtained by means of a hot-wire anemometer mounted on a sting downstream of the wing tips. The hot wire was held in alignment with a radius of the vortex, so that true absolute value of the velocity vector was measured, and errors due to the inherent directional sensitivity of the instrument were avoided. (This explains the velocity peak just outside the core in Fig. 1c.)

Results

Typical results of velocity-profile measurements are shown in Fig. 1. The profiles of absolute steady-state velocity $u(r, x)$ are shown for several streamwise distances x (measured downstream from the wing trailing edges) and for several values of uniform flow speed $U(x)$ outside the vortex.

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¹ Gibbons, J. E., "Vortex bursting in swirl flow," S.B. Thesis, Mass. Inst. Tech. (1962).

Linearized Interaction Curves for Plastic Beams under Combined Bending and Twisting

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It has been shown by Handelman¹ and by Hill² that for a prismatic beam of perfectly plastic material, subjected to combined bending and torsion, the plastic-stress function $\phi(x, y)$ at a given section can be determined by solving the differential equation

$$\frac{\partial}{\partial x} \left[\frac{y\phi_x}{(1 - \phi_x^2 - \phi_y^2)^{1/2}} \right] + \frac{\partial}{\partial y} \left[\frac{y\phi_y}{(1 - \phi_x^2 - \phi_y^2)^{1/2}} \right] \frac{\mu}{\lambda} = 0 \quad (1)$$

together with the condition $\phi_s = 0$ along the boundary. In Eq. (1), μ represents the ratio of the rate of twist to the rate of curvature, and λ is a constant depending on the yield condition assumed in the analysis. When the stress function ϕ throughout the section is known, the limiting combination of the bending moment M and the twisting moment T may be computed by the following expressions:

$$M = \sigma_0 \iint y(1 - \phi_x^2 - \phi_y^2)^{1/2} dx dy \quad (2)$$

$$T = 2\tau_0 \iint \phi dx dy \quad (3)$$

in which σ_0 and τ_0 are the yield stresses in simple tension and in shear, respectively.

Equation (1) is a nonlinear partial differential equation and can be solved numerically by Southwell's relaxation method. Recently Steel³ and Imegwu⁴ have obtained a number of solutions for circular, square, and triangular cross sections. Their results are summarized in a nondimensional form in Fig. 1. It is interesting to note that the plastic interaction between bending and torsion is virtually independent of the cross-sectional shape of the beam.

In solving practical problems, it is often convenient to use the so-called piecewise linear interaction curves that are derived from the true interaction curve through proper linearization. Obviously, the accuracy of the solution to a given problem depends directly on how close the linear interaction curves approximate the actual curve. Because of the lack of an exact solution, Sankaranarayanan and Hodge⁵ have suggested a two-segment linear approximation based on a lower-bound interaction curve. Unfortunately, this approximation, shown as the dotted line in Fig. 1, deviates appreciably from the numerical results obtained by Steele and Imegwu. A new type of linear approximation therefore is proposed herein. It consists of four linear segments AB,

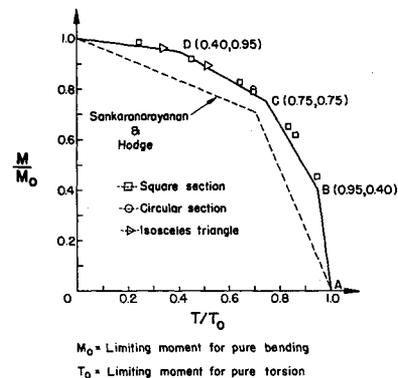


Fig. 1 Inelastic interaction curves for solid sections

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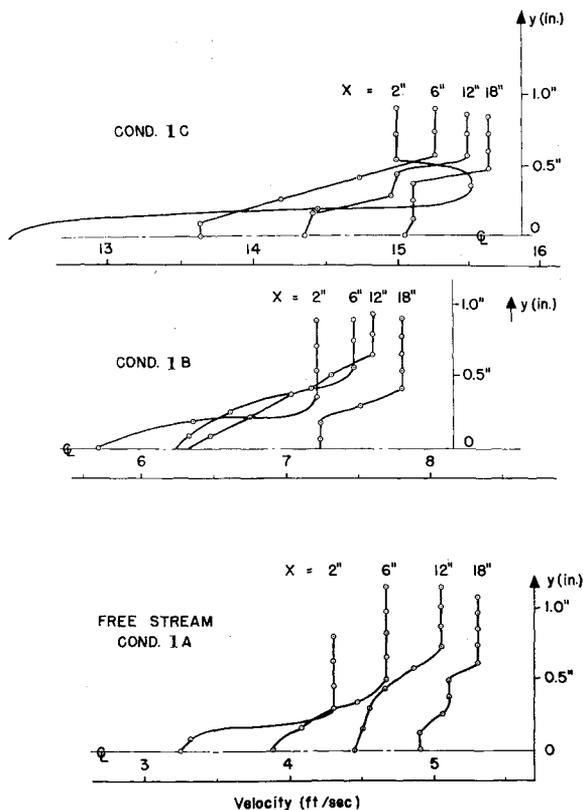


Fig. 1 Velocity profiles measured for three freestream velocities

The formation of several nearly discrete cylindrical shear regions is evident in Fig. 1. Under certain conditions of high speed, high angle of attack, and small wing-tip separation, as many as four such layers were obtained, as shown in Fig. 2.

The data shown in Fig. 2 could not be repeated from day to day, which may indicate a strong sensitivity of the phenomena to test parameters. Excellent qualitative consistency was observed, however, in all tests. The uncertainties and inaccuracies of the experiment were much too small to alter the shape of the velocity profiles.

Conclusions

A vortex wake has a tendency to form rather sharp piecewise cylindrical shear layers, which may or may not be unstable, depending on swirl velocity, stream velocity, and swirl profile. Similar phenomena can be observed in condensation trails from airplanes.

The phenomenon is related to similar phenomena occurring in flows in rotating containers. The region near the center of a vortex rotates almost as a solid body, whereas in the outer region the flow approaches a potential vortex flow.

The velocity field in the vortex core therefore may be governed by the effects of angular velocity, for example, as a possible tendency for the velocity field to be aligned with the vortex lines with a resulting modification of the flow stability. The data suggest that vortex bursting may not be a single phenomenon, but rather a sequency of quite orderly processes.

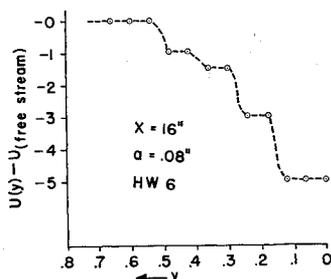


Fig. 2 Velocity profile showing four plateaus. More points than indicated were taken